# Geometry 3 - Miscellaneous

TSS Math Club

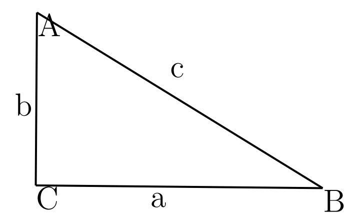
Nov 2022

## 1 Pythagorean Theorem

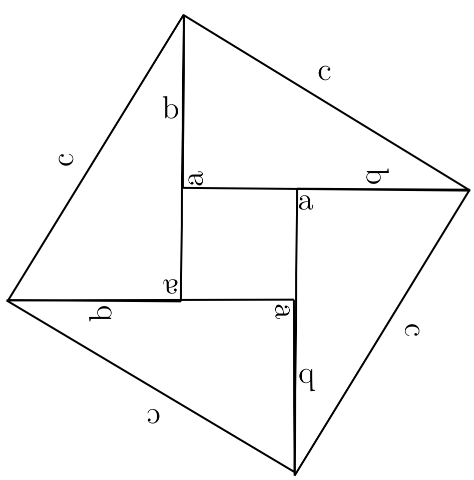
In a right-triangle,

$$a^2 + b^2 = c^2$$

where a and b are two sides and c is the hypotenuse.




### 1.1 Proof



## 2 Trigonometry

### 2.1 Definitions

Sine or $\sin(\theta)$ :

Cosine or $\cos(\theta)$ :

Tangent or $\tan(\theta)$ :

### 2.2 Pythagorean Theorem

$$\sin(\theta) + \cos(\theta) = 1$$

### 2.3 Triangle Area Formula with Sine

$$S = \frac{ab\sin C}{2}$$

#### 2.3.1 Proof

### 2.4 Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R = d$$

#### 2.4.1 Proof

### 2.5 Law of Cosines

$$c^2 = a^2 + b^2 - 2ab\cos C$$

#### 2.5.1 Proof

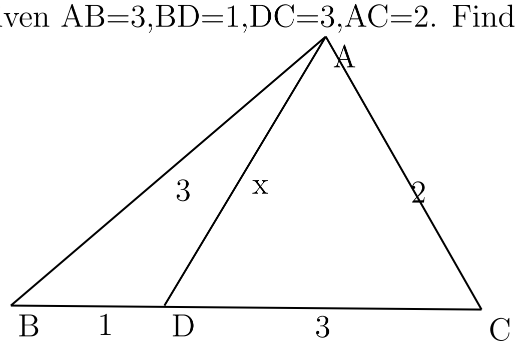
## 2.6 Problem

### 2.6.1 Heron's Formula

$$S = \sqrt{s(s-a)(s-b)(s-c)}, s = \frac{a+b+c}{2}$$

### 2.6.2 Problem

Given AB=3,BD=1,DC=3,AC=2. Find AD.



#### 2.6.3 Problem, Euclid 2022 Q8 b)

Consider the following statement:

> There is a triangle that is not equilateral whose side lengths form a geometric sequence, and the measures of whose angles form an arithmetic sequence.

Show that this statement is true by finding such a triangle or prove that it is false by demonstrating that there cannot be such a triangle.

# 3 Transversals

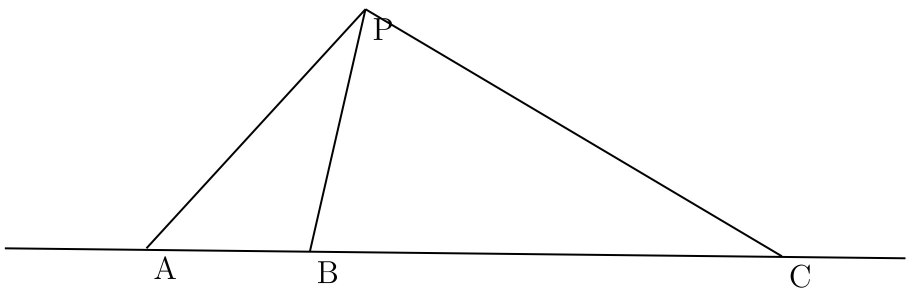
## 3.1 Directed Segments

Definition

## 3.2 Stewart's Theorem

If A,B,C collinear and P is any other point, then

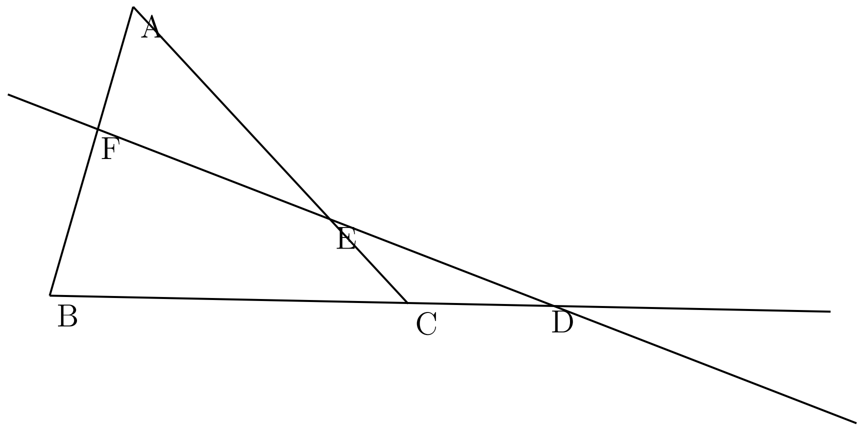
$$PA^2 \cdot BC + PB^2 \cdot CA + PC^2 \cdot AB + BC \cdot CA \cdot AB = 0$$



### 3.3 Menelaus' Theorem

Suppose we have a triangle ABC, and a transversal line that crosses BC, AC, and AB at points D, E, and F respectively, with D, E, and F distinct from A, B, and C, then

$$\frac{AF}{FB} \cdot \frac{BD}{DC} \cdot \frac{CE}{EA} = -1$$




### 3.4 Menelaus' Inverse Theorem

Suppose we have a triangle ABC with D on BC, E on AC, F on AB, such that,

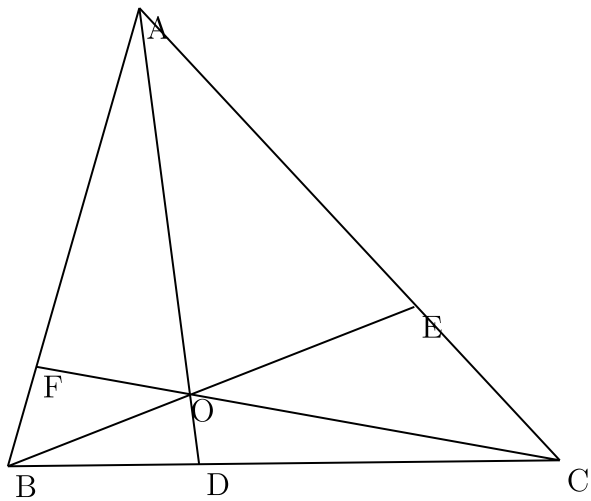
$$\frac{AF}{FB} \cdot \frac{BD}{DC} \cdot \frac{CE}{EA} = -1$$

then D,E, F collinear.

### 3.5 Ceva's Theorem

Given a triangle ABC, let the lines AO, BO and CO be drawn from the vertices to a common point O (not on one of the sides of ABC), to meet opposite sides at D, E and F respectively, then

$$\frac{AF}{FB} \cdot \frac{BD}{DC} \cdot \frac{CE}{EA} = 1$$




### 3.6 Ceva's Inverse Theorem

Suppose we have a triangle ABC with D on BC, E on AC, F on AB, such that,

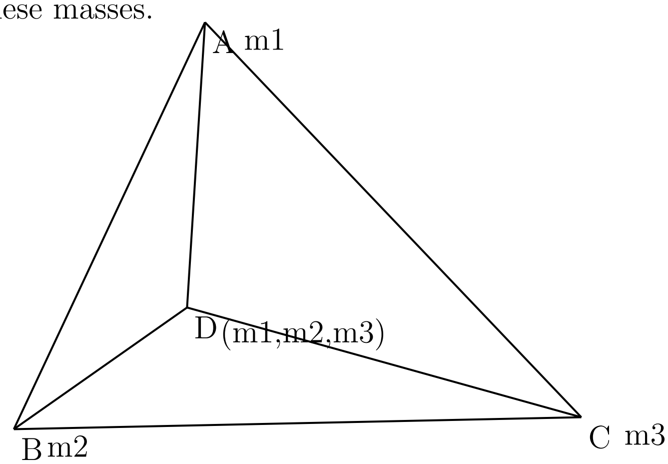
$$\frac{AF}{FB} \cdot \frac{BD}{DC} \cdot \frac{CE}{EA} = 1$$

then AD,BE,CF concurrent.

# 4 Barycentric Coordinate

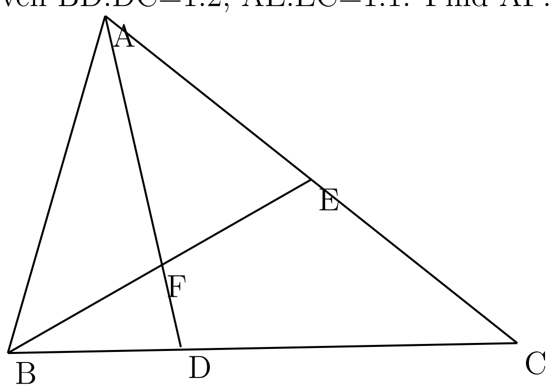
## 4.1 Definition

The barycentric coordinates of a point can be interpreted as masses placed at the vertices of the simplex, such that the point is the center of mass (or barycenter) of these masses.



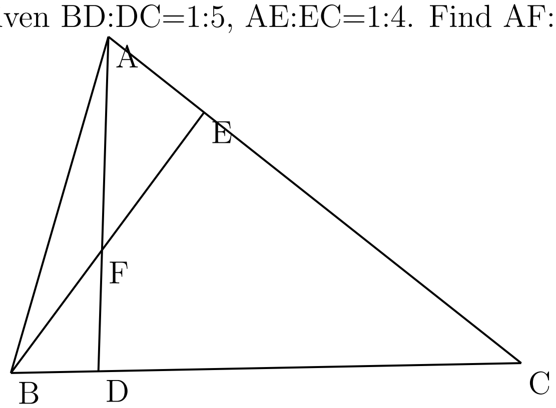

## 4.2 Example

Given BD:DC=1:2, AE:EC=1:1. Find AF:FD.




## 4.3 Problem

Given BD:DC=1:5, AE:EC=1:4. Find AF:FD.



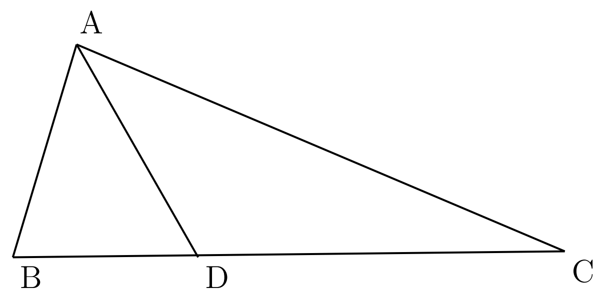
# 5 Angle Bisector

## 5.1 Definition

## 5.2 Angle Bisector Theorem

If AD bisects $\angle A$, then

$$\frac{BD}{CD} = \frac{AB}{AC}$$

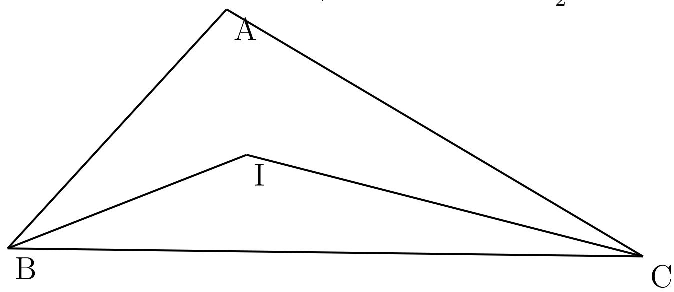



## 5.3 Theorem

Angle bisectors of a trinagle are concurrent, the point is called the incenter of the triangle

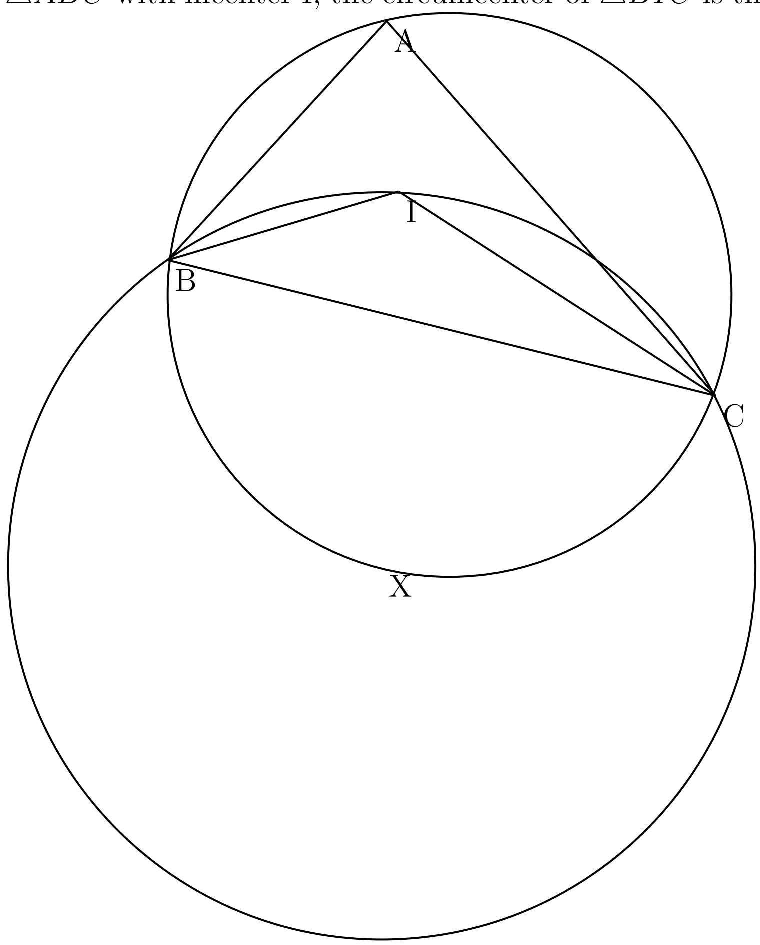
## 5.4 Theorem

In $\triangle ABC$ with incenter I, $\angle BIC = 90^\circ + \frac{1}{2}\angle A$




## 5.5 Theorem

In $\triangle ABC$ with incenter I, the circumcenter of $\triangle BIC$ is the mid point of the arc $\overset{\frown}{BC}$.



# 6 Median

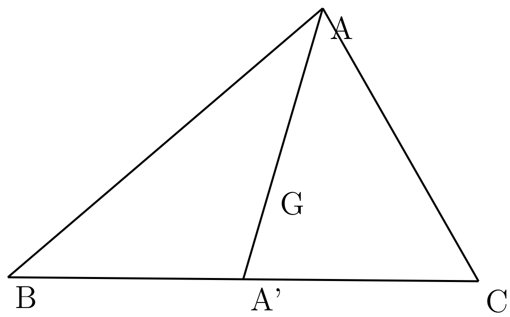
## 6.1 Definition

## 6.2 Theorem

Medians of triangle are concurrent. The point is called the centroid of the triangle.

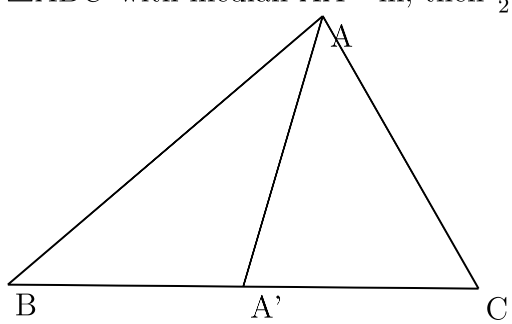
## 6.3 Theorem

In $\triangle ABC$ with centroid G and A' as the midpoint of BC, AG=2GA'.




## 6.4 Median Length Formula

In $\triangle ABC$ with median AA'=m, then $\frac{1}{2}m^2 = b^2 + c^2 - \frac{1}{2}a^2$

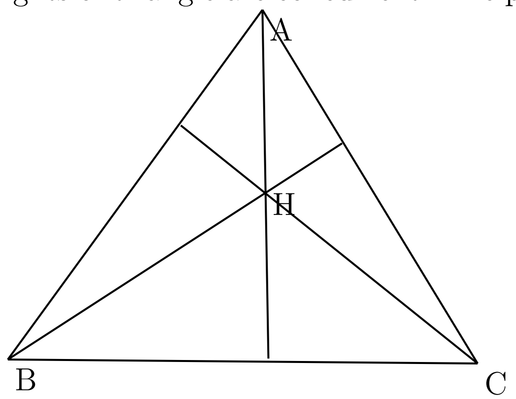


# 7 Height

## 7.1 Definition

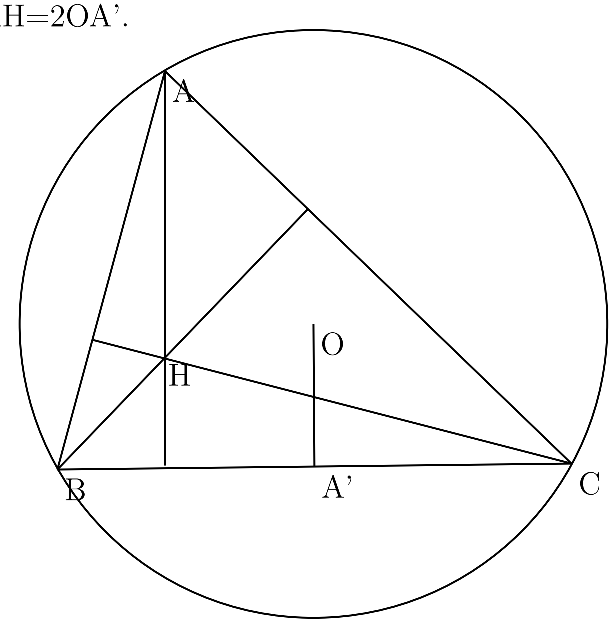
## 7.2 Theorem

Heights of triangle are concurrent. The point is called the orthocenter of the triangle.




## 7.3 Theorem

In $\triangle ABC$ with orthocenter H, A' the midpoint of BC, and the circumcenter O, AH=2OA'.



### 7.4 Theorem:

O the circumcenter,G the centroid,H the orthocenter are collinear. This line is called the Euler line of the triangle.